

Flavor Neutrino Masses giving $\sin\theta_{13}=0$

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Among neutrino mixings, the reactor mixing angle, θ_{13} , is observed to be almost vanishing and is consistent with $\theta_{13} = 0$. We discuss how the condition of $\theta_{13} = 0$ constrains models of neutrino mixings and show that, for flavor neutrino masses given by M_{ij} ($i, j=e, \mu, \tau$), two conditions of $M_{e\tau} = -e^{2i\gamma} \tan\theta_{23} M_{e\mu}$ and $M_{\tau\tau} = e^{4i\gamma} M_{\mu\mu} + e^{2i\gamma} (2/\tan 2\theta_{23}) M_{\mu\tau}$ lead to $\theta_{13}=0$, where θ_{23} is the atmospheric neutrino mixing angle and γ is its associated phase. The rephasing invariance can select two phases provided by $\alpha = \arg(M_{e\mu})$ and $\beta = \arg(M_{e\tau})$, giving $\gamma = (\beta - \alpha)/2$.

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Flavor neutrinos, $\nu_{e,\mu,\tau}$, are mixed with each other during their flight, where neutrinos are described by mass-eigenstates, $\nu_{1,2,3}$ [1]. There are three distinct neutrino mixings, the atmospheric neutrino mixing [2, 3], the solar neutrino mixing [4–6] and the reactor neutrino mixing [7], which are, respectively, denoted by three mixing angles, θ_{23} , θ_{12} and θ_{13} , for the ν_i - ν_j mixing ($i, j=e, \mu, \tau$). The masses $m_{1,2,3}$ and these mixing angles are currently constrained to be [8]:

$$\Delta m_{\odot}^2 [10^{-5} \text{eV}^2] = 7.65^{+0.23}_{-0.20}, \quad |\Delta m_{atm}^2| [10^{-3} \text{eV}^2] = 2.40^{+0.12}_{-0.11}, \quad (1)$$

where Δm_{atm}^2 , and Δm_{\odot}^2 are neutrino mass squared differences given by $\Delta m_{\odot}^2 = m_2^2 - m_1^2 (> 0)$ [9] for solar neutrinos, and $\Delta m_{atm}^2 = m_3^2 - m_1^2$ for atmospheric neutrinos, and

$$\sin^2 \theta_{12} = 0.304^{+0.022}_{-0.016}, \quad \sin^2 \theta_{23} = 0.50^{+0.07}_{-0.06}, \quad \sin^2 \theta_{13} = 0.01^{+0.016}_{-0.011}. \quad (2)$$

The neutrino mixings are parameterized by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) unitary matrix U_{PMNS} [1], which converts the massive neutrinos ν_i ($i = 1, 2, 3$) into the flavor neutrinos ν_f ($f = e, \mu, \tau$): $\nu_f = \sum_{i=1}^3 (U_{PMNS})_{fi} \nu_i$, and which is given by $U_{PMNS}^{PDG} = U_{\nu}^0 K^0$ with

$$U_{\nu}^0 = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta_{CP}} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix},$$

$$K^0 = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}), \quad (3)$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ ($i, j=1, 2, 3$), as adopted by the Particle Data Group (PDG) [10]. Leptonic CP violation is induced by one Dirac CP-violating phase (δ_{CP}) and three Majorana phases ($\phi_{1,2,3}$) [11], where the Majorana CP-violating phases are determined by two combinations of $\phi_{1,2,3}$ such as $\phi_i - \phi_1$ ($i=1, 2, 3$).

There are two distinct properties present in the observed data. One is that the mixing angle θ_{13} is suppressed to show $\sin^2 \theta_{13} \ll 1$. The other is that Δm_{atm}^2 and Δm_{\odot}^2 show the hierarchy $\Delta m_{\odot}^2 / |\Delta m_{atm}^2| \ll 1$. Since the data are consistent with $\sin\theta_{13} = 0$, a theoretical interest arises to find what conditions lead to $\sin\theta_{13} = 0$ [12]. There are known theoretical sources that lead to $\sin\theta_{13} = 0$, which include the μ - τ symmetry [13], the tri-bimaximal mixing scheme [14] and the strong scaling ansatz [15]. These examples call for specific relations among flavor neutrino masses. However, as far as the condition of $\theta_{13} = 0$ is concerned, they are over-constrained.

In this note, we consider minimum requirement on flavor neutrino masses to yield $\sin\theta_{13} = 0$. To do so, we use U_{PMNS} with three Dirac phases, δ , ρ and γ , and three Majorana phases $\varphi_{1,2,3}$, instead of U_{PMNS}^{PDG} to handle general

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phase structure of flavor neutrino mass matrix. Our U_{PMNS} is parameterized by U_ν and K [16] in place of U_ν^0 and K^0 :

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\gamma} & 0 \\ 0 & 0 & e^{-i\gamma} \end{pmatrix} \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{i\rho} & s_{13}e^{-i\delta} \\ -c_{23}s_{12}e^{-i\rho} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i(\delta+\rho)} & s_{23}c_{13} \\ s_{23}s_{12}e^{-i\rho} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i(\delta+\rho)} & c_{23}c_{13} \end{pmatrix},$$

$$K = \text{diag}(e^{i\varphi_1}, e^{i\varphi_2}, e^{i\varphi_3}). \quad (4)$$

The phases ρ and γ are redundant and can be removed by the phase redefinition. As a result, four phases in U_{PMNS}^{PDG} are given by $\delta_{CP} = \delta + \rho$ and $\phi_1 = \varphi_1 - \rho$ as well as $\phi_{2,3} = \varphi_{2,3}$. For the 2-3 rotation, one may choose the similar phase (τ) to ρ and δ , contributing to δ_{CP} as $\delta_{CP} = \delta + \rho + \tau$. However, we have proved that γ is a suitable phase for the 2-3 rotation [16]. The phase τ can be removed by introducing a new definition: $\rho' = \rho + \tau/2$, $\gamma' = \gamma + \tau/2$ and $\delta' = \delta + \tau/2$ as well as $\varphi'_2 = \varphi_2 - \tau/2$ and $\varphi'_3 = \varphi_3 + \tau/2$. As a result, we end up with the same definition of δ_{CP} : $\delta_{CP} = \delta' + \rho'$. Therefore, the parameterization with δ' , ρ' , and γ' gives Eq.(4) as a general form of U_{PMNS} .

Let M_ν be a flavor neutrino mass matrix parameterized by

$$M_\nu = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}, \quad (5)$$

on the $(\nu_e, \nu_\mu, \nu_\tau)$ -basis. Since $U_{PMNS}^T M_\nu U_{PMNS} = \text{diag}(m_1, m_2, m_3)$, it is not difficult to derive

$$\begin{aligned} & (e^{i(\rho-\delta)}a - e^{i(\rho+\delta)}\lambda_3) \sin 2\theta_{13} + 2y \cos 2\theta_{13} = 0, \\ & (\lambda_1 - \lambda_2) \sin 2\theta_{12} + 2x \cos 2\theta_{12} = 0, \\ & e \cos 2\theta_{23} - \frac{e^{-2i\gamma}f - e^{2i\gamma}d}{2} \sin 2\theta_{23} + e^{-i(\rho+\delta)}x \sin \theta_{13} = 0, \end{aligned} \quad (6)$$

as three vanishing off-diagonal elements [17], where

$$\begin{aligned} \lambda_1 &= e^{2i\rho} \frac{c_{13}^2 a - s_{13}^2 e^{2i\delta} \lambda_3}{c_{13}^2 - s_{13}^2}, \quad \lambda_2 = c_{23}^2 e^{2i\gamma} d + s_{23}^2 e^{-2i\gamma} f - 2s_{23}c_{23}e, \\ \lambda_3 &= s_{23}^2 e^{2i\gamma} d + c_{23}^2 e^{-2i\gamma} f + 2s_{23}c_{23}e, \end{aligned} \quad (7)$$

$$x = \frac{c_{23}e^{i(\rho+\gamma)}b - s_{23}e^{i(\rho-\gamma)}c}{c_{13}}, \quad y = s_{23}e^{i(\rho+\gamma)}b + c_{23}e^{i(\rho-\gamma)}c. \quad (8)$$

If $\sin \theta_{13} = 0$ is realized in Eq.(6), we obtain that

$$c = -e^{2i\gamma}t_{23}b, \quad (9)$$

$$f = e^{4i\gamma}d + e^{2i\gamma} \frac{1 - t_{23}^2}{t_{23}} e, \quad (10)$$

where $t_{23} = \tan \theta_{23}$. The phase γ turns out to be a phase difference between b and c . We finally reach

$$M_\nu = \begin{pmatrix} a & e^{i\alpha}|b| & -e^{i\beta}t_{23}|b| \\ e^{i\alpha}|b| & d & e \\ -e^{i\beta}t_{23}|b| & e & e^{2i(\beta-\alpha)}d + e^{i(\beta-\alpha)} \frac{1-t_{23}^2}{t_{23}} e \end{pmatrix}, \quad (11)$$

which provides the general structure of M_ν with $\sin \theta_{13} = 0$ [18], where α and β , respectively, stand for phases of b and c , determining

$$\gamma = \frac{\beta - \alpha}{2}. \quad (12)$$

The rephasing invariance of M_ν allows us to set a , d and e to be real numbers. Furthermore, d is taken to be positive without loss of generality.

One may wonder why Eqs.(9) and (10), which depend on the redundant phase γ , are appropriate. It is because b , c , d and f in M_ν is, respectively, transformed into $e^{i(\rho+\gamma)}b$, $e^{i(\rho-\gamma)}c$, $e^{2i\gamma}d$ and $e^{-2i\gamma}f$ after the redundant phases in U_{PMNS} are removed. Namely, for U_{PMNS}^{PDG} , the corresponding mass parameters get modified into $b^{PDG} = e^{i(\rho+\gamma)}b$, $c^{PDG} =$

$e^{i(\rho-\gamma)}c$, $d^{PDG} = e^{2i\gamma}d$, $e^{PDG} = e$ and $f^{PDG} = e^{-2i\gamma}f$ as obvious notations, which, give $y = s_{23}b^{PDG} + c_{23}c^{PDG}$ and $f^{PDG} = d^{PDG} + \frac{1-t_{23}^2}{t_{23}}e^{PDG}$ without the apparent dependence of γ . See Ref.[17] for more details. One may also wonder if $\tan 2\theta_{12}$ given by Eq.(6) yielding

$$\tan 2\theta_{12} = \frac{2e^{i(\rho+\frac{\alpha+\beta}{2})}}{c_{23}} \frac{|b|}{e^{i(\beta-\alpha)}|d| - t_{23}\kappa_e|e| - e^{2i\rho}\kappa_a|a|}, \quad (13)$$

remains real, where $\kappa_{a,e}$ take care of the sign of a and e . Namely, phases must be cancelled out each other. Since Eq.(13) contains the phase ρ , we have to express ρ in terms of the flavor neutrino masses. It is convenient to use the relation [17] given by

$$\rho = \arg(X), \quad (14)$$

where X is an analog of x in Eq.(8) but is determined by $U_{PMNS}^\dagger M_\nu^\dagger M_\nu U_{PMNS} = \text{diag}(m_1^2, m_2^2, m_3^2)$. The parameter X is given by

$$X = \frac{c_{23}e^{i\gamma}B - s_{23}e^{-i\gamma}C}{c_{13}}, \quad (15)$$

where B and C are defined in

$$M_\nu^\dagger M_\nu = \begin{pmatrix} A & B & C \\ B^* & D & E \\ C^* & E^* & F \end{pmatrix}. \quad (16)$$

Since X is calculated to be

$$X = \frac{e^{i\frac{\alpha+\beta}{2}}|b|}{c_{23}} \left[\kappa_a|a| + e^{-i(\alpha+\beta)} \left(e^{i(\beta-\alpha)}|d| - t_{23}\kappa_e|e| \right) \right], \quad (17)$$

one may replace $e^{i(\beta-\alpha)}|d| - t_{23}\kappa_e|e|$ in Eq.(13) by X and we reach the following expression of $\tan 2\theta_{12}$:

$$\tan 2\theta_{12} = \frac{2}{c_{23}} \frac{|b|}{\frac{c_{23}e^{-i\rho}X}{|b|} - 2\cos\left(\rho - \frac{\alpha+\beta}{2}\right)\kappa_a|a|}. \quad (18)$$

Since $\rho = \arg(X)$, we prove that $\tan 2\theta_{12}$ certainly remains real and that our consideration using Eq.(4) is correct.

The μ - τ symmetric case, which provides $\theta_{13} = 0$ and $\theta_{23} = \pi/4$, corresponds to $t_{23} = 1$ in Eq.(11) leading to

$$M_\nu = \begin{pmatrix} a & e^{i\alpha}|b| & -e^{i\beta}|b| \\ e^{i\alpha}|b| & d & e \\ -e^{i\beta}|b| & e & e^{2i(\beta-\alpha)}d \end{pmatrix}, \quad (19)$$

as well as $\alpha = \beta$. It should be noted that, if $\alpha \neq \beta$, the μ - τ symmetry is broken but the prediction of $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ remains intact. This mass matrix is invariant under the interchange of $\nu_\mu \leftrightarrow -e^{i(\beta-\alpha)}\nu_\tau$. This is the extended μ - τ symmetry [19], which naturally manifests itself in a special case of Eqs.(9) and (10). The tri-bimaximal mixing scheme further predicts $\sin^2 \theta_{12}$ to be 1/3 in Eq.(6). The strong scaling ansatz is recovered by $e = -e^{i(\beta-\alpha)}t_{23}d$ leading to

$$M_\nu = \begin{pmatrix} a & e^{i\alpha}|b| & -e^{i\beta}t_{23}|b| \\ e^{i\alpha}|b| & d & -e^{i(\beta-\alpha)}t_{23}d \\ -e^{i\beta}t_{23}|b| & -e^{i(\beta-\alpha)}t_{23}d & e^{2i(\beta-\alpha)}t_{23}^2d \end{pmatrix}. \quad (20)$$

Another similar but new type of M_ν is given by $e = e^{i(\beta-\alpha)}d/t_{23}$ leading to

$$M_\nu = \begin{pmatrix} a & e^{i\alpha}|b| & -e^{i\beta}t_{23}|b| \\ e^{i\alpha}|b| & d & e^{i(\beta-\alpha)}d/t_{23} \\ -e^{i\beta}t_{23}|b| & e^{i(\beta-\alpha)}d/t_{23} & e^{2i(\beta-\alpha)}d/t_{23}^2 \end{pmatrix}. \quad (21)$$

In summary, there are three important results found in our discussions:

- two conditions on M_ν to give $\sin\theta_{13} = 0$ consisting of

$$M_{e\tau} = -e^{2i\gamma}t_{23}M_{e\mu}, \quad M_{\tau\tau} = e^{4i\gamma}M_{\mu\mu} + e^{2i\gamma}\frac{1-t_{23}^2}{t_{23}}M_{\mu\tau}, \quad (22)$$

where M_{ij} ($i, j=e, \mu, \tau$) is an i - j matrix element of M_ν ,

- the “natural” emergence of the extended μ - τ symmetry that arises from the requirement of $M_{e\tau} = -e^{2i\gamma}M_{e\mu}$ and $M_{\tau\tau} = e^{4i\gamma}M_{\mu\mu}$ as in Eq.(19), where the phase γ breaks the exact μ - τ symmetry, and
- Eq.(21) as a new type of M_ν derived by $M_{\mu\tau} = e^{2i\gamma}M_{\mu\mu}/t_{23}$, which is similar to M_ν with $M_{\mu\tau} = -e^{2i\gamma}t_{23}M_{\mu\mu}$ for the strong scaling ansatz.

Since the appearance of the Dirac CP-violation requires $\sin\theta_{13} \neq 0$, we may add the following terms to M_ν to break the relations Eqs.(9) and (10):

$$\Delta M_\nu = \begin{pmatrix} 0 & 0 & \delta c \\ 0 & 0 & 0 \\ \delta c & 0 & \delta f \end{pmatrix}. \quad (23)$$

These presumably small parameters yield a nonvanishing value of y , thereby, giving $s_{13} \neq 0$ as follows:

$$\delta c = e^{-i(\rho-\gamma)}\frac{y}{c_{23}}, \quad \delta f = e^{-i(\delta+\rho-2\gamma)}\frac{2s_{13}x}{\sin 2\theta_{23}}. \quad (24)$$

We will consider phenomenological analysis based on $M_\nu + \Delta M_\nu$ and see effects of the leptonic CP violation in our future publication [20].

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